Camera calibration

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1 Classical camera calibration theory

In classical camera calibration theory, one needs to solve the following matrix equation with 11 unknowns and 2N x 11 knowns.

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -u_2X_2 & -u_2Y_2 & -u_2Z_2 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -v_2X_2 & -v_2Y_2 & -v_2Z_2 \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{bmatrix} \cdot \begin{bmatrix} q_{11} \\ q_{12} \\ q_{13} \\ q_{14} \\ q_{21} \\ q_{22} \\ q_{23} \\ q_{24} \\ q_{31} \\ q_{32} \\ q_{33} \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ . \\ . \\ . \\ . \\ . \\ u_N \\ v_N \end{bmatrix}$$

To solve this equation, the pixel and real world coordinates of at least six points must be known[1] ($N >= 6$). (X_i, Y_i, Z_i) is the world coordinates with (u_i, v_i) image coordinates. q_{ij} is the unknown camera calibration constants.

2 Reconstruction of 3D coordinates

In stereo vision, when the camera calibration for both cameras are known ($C = [q_{ij}]$ and $C' = [q'_{ij}]$), the following equation will give the real world coordinates (X, Y, Z) of a common scene point in projected into camera coordinates (u, v) and (u', v').

$$\begin{bmatrix} q_{11} - uq_{31} & q_{12} - uq_{32} & q_{13} - uq_{33} \\ q_{21} - vq_{31} & q_{22} - vq_{32} & q_{23} - vq_{33} \\ q'_{11} - u'q'_{31} & q'_{12} - u'q'_{32} & q'_{13} - u'q'_{33} \\ q'_{21} - v'q'_{31} & q'_{22} - v'q'_{32} & q'_{23} - v'q'_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} u - q_{14} \\ v - q_{24} \\ u' - q'_{14} \\ v' - q'_{24} \end{bmatrix}$$

[1] Source: UWA Computer Vision IT412 lecture notes